

$$y' = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$u \frac{dv}{dx} + v \frac{du}{dx} + \frac{1-2x}{x^2} uv = 1$$

$$u \left[\frac{dv}{dx} + \frac{1-2x}{x^2} v \right] + v \frac{du}{dx} = 1$$

$$\frac{dv}{dx} + \frac{1-2x}{x^2} v = 0$$

$$\frac{1}{v} dv = \int \frac{1}{x^2} dx + 2 \int \frac{1}{x} dx$$

$$\sin^2 x + \cos^2 x = 1$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x}$$

$$\operatorname{ctg} x = \frac{\cos x}{\sin x}$$

$$\operatorname{tg} x \operatorname{ctg} x = 1$$

$$\sin 2x = 2 \sin x \cos x$$

$$3 \log_7 x + \frac{1}{5} \log_7 b - (x+2) \log_7 3$$

$$\log_7 x^3 + \log_7 \sqrt[5]{b} - \log_7 3^{(x+2)}$$

$$\log_7 x^{\frac{3}{5}} - \log_7 3^{(x+2)}$$

$$\log_7 \frac{x^{\frac{3}{5}}}{3^{(x+2)}}$$

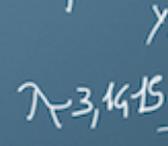
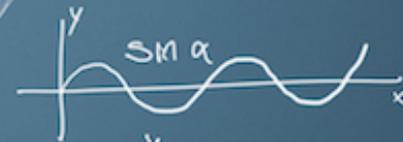
$$3x^3 - 5x^2 + 2x = 0$$

$$x \cdot (3x^2 - 5x + 2) = 0$$

$$x = 0 \vee 3x^2 - 5x + 2 = 0$$

$$x_1 = 0 \vee x_{2,3} = \frac{5 \pm \sqrt{25 - 24}}{6}$$

$$x_1 = 0 \quad x_2 = 1 \quad x_3 = \frac{2}{3}$$



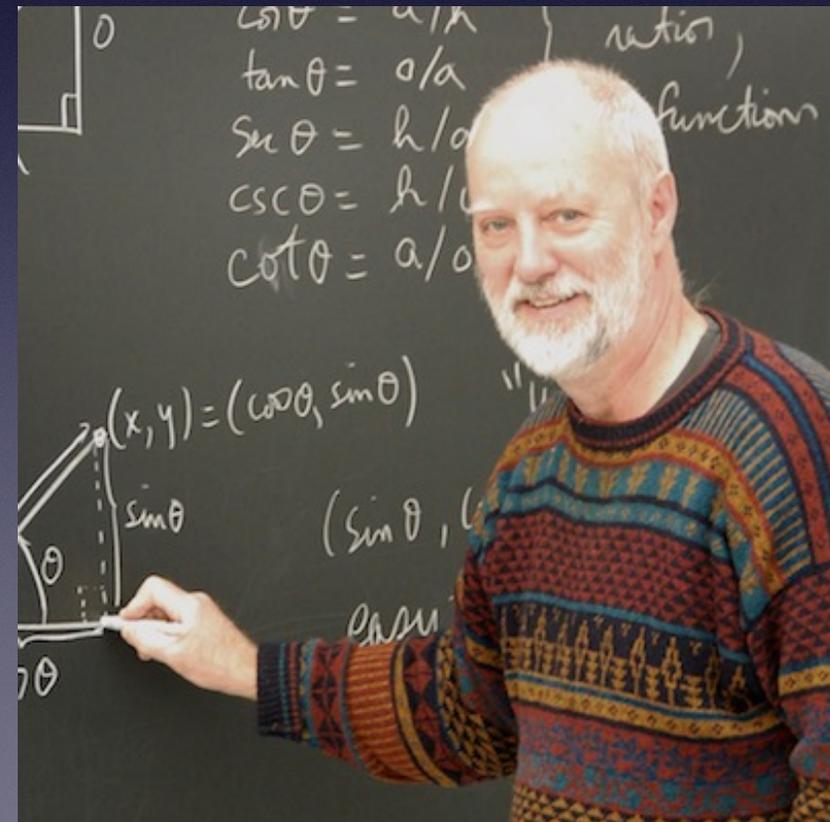
Four-Day Summer Math Bridging for STEM Success

John Amanatides

Bethune College
York University

Kim Maltman

Department of Mathematics
& Statistics
York University



Our Context

- Large urban commuter university (55K students)
- college-based system (similar to Oxbridge)
- colleges help make a big university feel small

Bethune College

- all Science and Engineering students (1,700 new, 6,900 total) are part of Bethune College
- We provide orientation, academic, co-curricular, and extra-curricular opportunities/events, support
- Student government provides social events, intramural sports

Typical Student

- First in family at university
- Average marks
- Commuter
- Part-time work
- Career-oriented

Math Prerequisites

- for most programs typically two grade 12 maths:
 - Advanced Functions
 - Calculus and Vectors

First-Year Math

- MATH 1013/1014: Applied Calculus for Physical Sciences/Engineering
- MATH 1300/1310: Calculus with Applications for Math & CS majors
- MATH 1505: Math for Life and Social Sciences for Biology, Psychology, etc
- MATH 1019: Discrete Math for CS

How Students Do (2012-13)

- MATH 1013 3.0: Drop + Fail = 31%
- MATH 1014 3.0: Drop + Fail = 36% (only 43% pass both)
- MATH 1019 3.0: Drop + Fail = 56%
- MATH 1300 3.0: Drop + Fail = 34%
- MATH 1310 3.0: Drop + Fail = 27 % (only 48% pass both)
- MATH 1505 6.0: Drop + Fail = 36%

Source: OIRA

Not Acceptable

- a Math prof (Kim) analyzed how students answered exam questions
- realized basic algebraic and trigonometric skills were lacking
- many times because of the way these topics were typically taught in high schools

High School Math

- many students taught to apply formulas in mechanical, rote manner
- relied on memorizing sequences of manipulations
- “template” problems, minimally changed for tests
- little understanding of what is going on

Pilot in Science

- Kim started developing diagnostic test in 2005
- developed four-day intervention that concentrated on algebraic and trigonometric understanding in 2005/06
- tracked cohort over several years
- success: drop-out rates cut almost 50%
- success continued into 2nd-year math

Math Background Quiz

- 50 multiple choice questions
- online, told not to use calculator
- can take quiz up to 3 times
- results go directly to student on test completion and also go to enrolment advisor

Diagnostic Emphasis

- incoming students informed quiz is diagnostic only, doesn't affect their acceptance
- enrolment advisor can recommend Math course to take but student decides

Typical Questions

Question 6:

$\cos(225^\circ)$

1. is > 0
2. is < 0
3. is equal to 0
4. which of 1, 2 or 3 is correct cannot be determined without a calculator

Question 10:

The sine of $\pi/2$ radians, $\sin(\pi/2)$

1. is equal to 1
2. is equal to 0
3. is equal to $1/2$
4. is undefined
5. cannot be determined without the use of a calculator

Question 12:

It is possible to find an angle θ such that $\sin(\theta) = 6/5$

- True
- False

Typical Questions

Question 23:

$(x+t)(y-z)$ is equal to

1. $xy - tz$
2. $xy + yt - zt$
3. $xy - xz - zt$
4. $y(x+t) - z(x+t)$
5. None of the previous

Question 21:

If x and y are not both equal to zero, $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$

- True
 False

Question 26:

A person with \$2.05 in change, consisting entirely of 10-cent and 25-cent pieces, has three more 10-cent pieces than 25-cent pieces. If we let x be the number of 10-cent pieces, an equation which correctly incorporates the above information is

1. $25(x-3) + 10x = 205$
2. $205/(10x) = 25(x-3)$
3. $10(x-3) + 25x = 205$
4. None of the previous

Math Background Tutorial

- voluntary
- 4 days long, 4 hours/day, 16 hrs total
- held in each of 3 weeks before Frosh Week
- 20 students/instructor (math faculty member)
- students go over worksheets and get iterative conceptual feedback from instructor
- Free!

Worksheets

- typically no more than 10 questions in block
- concentrate on one conceptual point
- students who fail to answer all questions get opportunity to listen to conceptual explanation of underlying ideas and go back to retry questions
- possibly multiple iterations of block

Worksheet questions not “marked”

- each question True/False
- instructor says:
 - “all answered correctly, go to next block”
 - “there is only one error (probably carelessness)”
direct student to go over 3-4 questions
 - “there is more than one error”; goes over
concept

TRIG BACKGROUND EXERCISES

NO CALCULATORS!!!

TRUE OR FALSE QUESTIONS: For each question below circle **T** if the statement is always true and **F** if there are ANY circumstances in which it is false (even if there are some special cases where it happens to be true).

I. THE MEANING OF “RADIAN” MEASURE FOR ANGLES

In this section, “Quadrant I”, “Quadrant II”, “Quadrant III” and “Quadrant IV” label the four quadrants of the xy coordinate system in the conventional manner, i.e., with Quadrant I being the upper right quadrant, Quadrant II the upper left quadrant, Quadrant III the lower left quadrant and Quadrant IV the lower right quadrant. All angles are given in radians. Quadrants I, II, III and IV are also sometimes called “the first quadrant”, “the second quadrant”, “the third quadrant” and “the fourth quadrant”, respectively.

- | | | |
|---|---|---|
| 1. The angle $\theta = 5\pi/8$ lies in Quadrant I. | T | F |
| 2. The angle $\phi = -3\pi/8$ lies in Quadrant I. | T | F |
| 3. The angle $z = 5\pi/4$ lies in the third quadrant. | T | F |
| 4. The angle $q = 13\pi/3$ lies in Quadrant IV. | T | F |
| 5. The angle $\theta = -14\pi/5$ lies in Quadrant III. | T | F |
| 6. The angle $9\pi/5$ lies in the fourth quadrant. | T | F |
| <hr/> | | |
| 7. The angles $9\pi/2$ and $\pi/2$ describe the <i>same</i> angular position. | T | F |
| 8. The angles $w_1 = -5\pi/3$ and $w_2 = \pi/3$ describe the <i>same</i> angular position. | T | F |
| 9. The angles $w_1 = 4\pi/3$ and $w_2 = 14\pi/3$ describe the <i>same</i> angular position. | T | F |
| 10. The angles $w_1 = 21\pi/5$ and $w_2 = -4\pi/5$ describe the <i>same</i> angular position. | T | F |
| 11. The angles $15\pi/4$ and $-9\pi/4$ describe the <i>same</i> angular position. | T | F |
| 12. The angles $w_1 = 8\pi/5$ and $w_2 = 2\pi/5$ describe the <i>same</i> angular position. | T | F |

IV: The idea of a “solution” to an algebraic equation

- | | | |
|---|---|---|
| 1. $x = 1$ is the <i>only</i> real number solution of $2x^2 - x - 1 = 0$. | T | F |
| 2. $x = 3$ is the <i>only</i> real number solution of $x^2 - 6x + 9 = 0$. | T | F |
| 3. There are no real solutions of the equation $x^2 + 2 = 0$. | T | F |
| 4. $x = 2$ is a solution of $x^3 - 3x^2 + 3x = 2$. | T | F |
| 5. $u = \sqrt{\frac{\pi}{2}} - 1$ is a solution of the equation $\cos(u^2 + 1) = 1$. | T | F |
| 6. $x = 0$ is a solution of the equation $\sqrt{3 \cos(x) + \sec[\sin(x)]} = 2$ | T | F |
| 7. There are no real number solutions of the equation $x^4 + 2x^2 + 5 = 0$. | T | F |
| <hr/> | | |
| 8. If a, b are constants which are such that $t = 2$ is a solution of the equation $at + b = 0$, then $t = 4$ is a solution of the related equation $2at + 2b = 0$. | T | F |
| 9. If $f(x)$ is any function defined for all x with $0 < x < 4$, then $x = 2$ is <i>necessarily</i> a solution of the equation $f(x) - f(x^2 - 2) = 0$. | T | F |
| 10. If $f(x)$ is any function defined for all x , with $f(3) = 1$, then $x = 1$ is <i>necessarily</i> a solution of the related equation $f(3 \cos(x^2 - 1)) = 1$. | T | F |
| 11. The equation $\sin(2x) = 1/2$ has no real number solutions. | T | F |
| 12. The equation $\tan(2x + 1) = 1$ has no real number solutions. | T | F |
| 13. The equation $2 + \tan(x^2 + 1) = 0$ has no real number solutions. | T | F |
| 14. The equation $\cos(x) = x$ has no solution. | T | F |

Results 2012-13

Course	BT Attendees			non-attendees		
	[#]	drop+fail%	GPA	[#]	drop+fail%	GPA
MATH 1013	[68]	17.6	4.0	[775]	45.5	3.5
MATH 1014	[64]	26.5	4.4	[598]	49.2	3.6
MATH 1021	[25]	20.0	5.2	[309]	46.3	4.0
MATH 1025	[20]	35.0	4.1	[337]	48.1	3.3
MATH 1019	[20]	55.0	2.9	[448]	57.6	2.6
MATH 1131	[32]	21.9	5.4	[428]	52.8	3.8
MATH 1200	[20]	30.0	4.4	[123]	38.2	4.2
MATH 1300	[30]	26.7	4.8	[439]	45.6	4.0
MATH 1310	[22]	27.3	3.7	[260]	38.5	4.2
MATH 1505	[58]	20.7	4.9	[1176]	48.0	4.2
MATH 1510	[19]	21.1	4.8	[484]	57.9	3.9
MATH 1520	[24]	20.8	5.4	[343]	48.1	4.3
MATH 2015	[19]	21.1	4.4	[226]	37.6	4.0

Results 2013-14

COURSE No.	BT attendees			non-attendees		
	[#]	drop+fail%	(GPA)	[#]	drop+fail%	(GPA)
MATH 1013	[51]	27.5	(4.4)	[980]	40.2	(3.9)
MATH 1014	[49]	26.5	(4.0)	[699]	36.7	(4.0)
MATH 1019	[27]	22.2	(3.9)	[485]	41.3	(3.7)
MATH 1021	[20]	20.0	(4.2)	[323]	44.9	(4.0)
MATH 1025	[27]	18.5	(5.5)	[539]	37.5	(4.4)
MATH 1090	[21]	28.6	(6.0)	[290]	35.2	(4.8)
MATH 1131	[33]	18.2	(5.4)	[572]	43.9	(3.9)
MATH 1200	[14]	28.6	(4.3)	[122]	39.3	(3.5)
MATH 1300	[26]	19.2	(5.3)	[497]	40.0	(4.1)
MATH 1310	[28]	35.7	(4.4)	[440]	48.6	(3.6)
MATH 1505	[65]	30.8	(3.6)	[1470]	54.6	(3.7)
MATH 1510	[19]	52.6	(3.0)	[478]	60.9	(3.9)
MATH 1520	[18]	16.7	(5.0)	[368]	41.6	(4.9)

Results 2014-15

COURSE No.	BT attendees			non-attendees		
	[#]	drop+fail%	(GPA)	[#]	drop+fail%	(GPA)
MATH 1013	[44]	25.0	(5.0)	[1227]	41.4	(3.8)
MATH 1014	[49]	30.6	(4.3)	[829]	49.6	(3.7)
MATH 1019	[16]	37.5	(2.9)	[594]	54.1	(3.2)
MATH 1021	[21]	19.0	(5.2)	[411]	48.7	(4.4)
MATH 1025	[26]	7.7	(6.0)	[746]	31.1	(4.6)
MATH 1090	[13]	15.4	(4.2)	[316]	42.4	(4.6)
MATH 1131	[26]	34.6	(5.0)	[549]	47.4	(3.8)
MATH 1300	[18]	50.0	(2.6)	[522]	44.6	(4.0)
MATH 1310	[24]	66.7	(3.4)	[494]	55.9	(3.4)
MATH 1505	[57]	26.3	(4.8)	[1618]	56.9	(4.1)
MATH 1510	[11]	45.5	(5.6)	[473]	66.0	(3.9)
MATH 1520	[15]	13.3	(5.6)	[284]	42.6	(4.4)

Results 2015-16

Course	Background Tutorial attendees			Non Attendees		
	Initial #	Drop_fail %	GPA	Initial #	Drop+fail %	GPA
1013	60	23.3	5.22	1292	43.4	3.91
1019	25	28.0	4.30	617	44.6	3.36
1021	14	28.6	3.21	415	41.9	3.97
1131	17	23.5	4.00	560	44.5	4.30
1300	35	20.0	4.28	576	32.1	4.21
1505	52	36.5	5.08	1314	51.8	3.60
1510	15	33.3	3.79	373	60.1	3.71

Biased towards High Achievers?

- No!
- students who took Math Background Tutorials on average did 10% worse on Background Quiz
- distribution of performance on Background Quiz uniformly shifted lower for students who attended sessions

Works well with Early Alert

- one year we had Math Background Tutorials during the October Reading Week
- tutorials were strongly promoted in one section of Math 1013 before Reading Week
- those who attended did, on average, 3% worse on Test 1 but were a factor of 2 less likely to drop or fail and did 5% better on final exam

Incoming Students Today

- all incoming Engineering and Science students nominally required to take Math Background Quiz before Enrolment Advising
- If low score encouraged to take Math Background Tutorials in weeks before Frosh Week
- Voluntary, Free
- Tutorials repeated in April before summer term

Resources

- faculty member does 3 tutorials = 1 semester course
- part-time instructor \$2,800 -> \$140/student

Going Forward

- Faculty of Health: recommends to weak students
- Engineering exploring on-line approaches:
 - Khan Academy videos
 - Pearson's Mastering Math
 - Math Background Tutorials are the "Gold Standard"

Questions?

amana@yorku.ca, kmaltman@yorku.ca

bethune.yorku.ca

bethune.yorku.ca/talks